Tutorial on Cognitive Logics

Mechanisms Predicting Human Inference Patterns

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Outline

Why are cognitive models of human thinking relevant?

- Smart devices, AI systems do (rarely) adapt to a specific users information process
  - They lack a theory of mind
- Tutorial systems rarely predict which errors you will do
- Human thinking is not yet understood, it is not transferable to systems

A word about logic

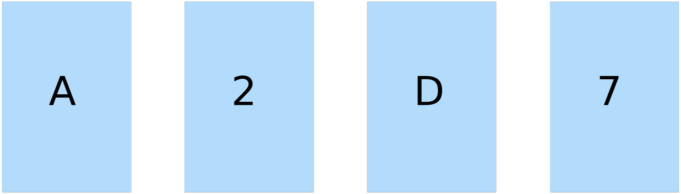
- Many logics for many purposes have been developed in AI, philosophy, math
  - They represent *correct* reasoning
- Change of perspective:
  - From: Use formal inference systems as a norm for correct human behavior ($\rightarrow$ deviations of human reasoning)
  - To: Use human "commonsense" reasoning to evaluate formal inference methods ($\rightarrow$ cognitive-adequacy of formalisms) or to check, how they need to be adapted.

Talk Overview

Section 1

Some Observations on the Human Reasoning Process

Observation 1: The Wason Selection Task [Was68]



- Given:
  - Four cards with a letter on one and a number on the other side
  - A rule to check: *If there is a vowel on one side then there is an even number on the other side of the card*
- Decide:
  - Exactly which cards to turn in order to check that the rule holds?

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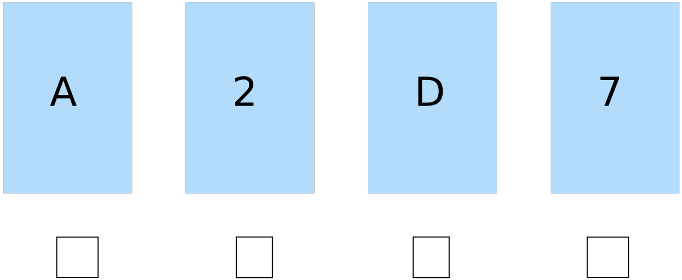

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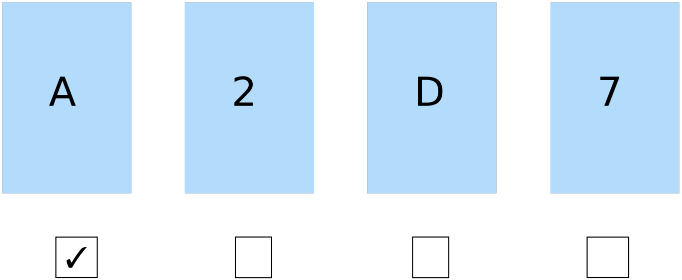

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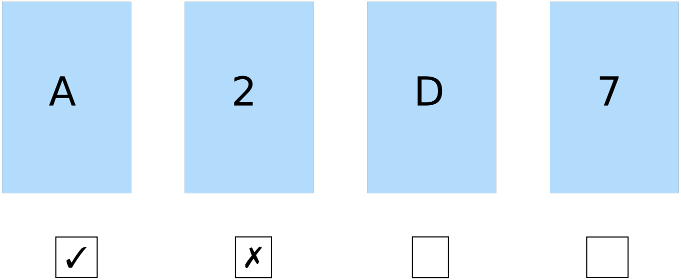

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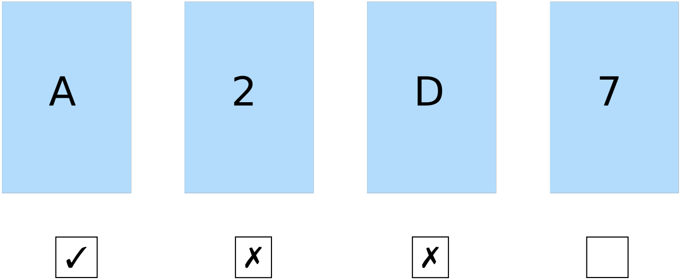

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✓	✗	✗	✓

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A	2	D	7
✓	✗	✗	✓

A rule: *If a vowel is on one side then an even number is on the other side*

Percentage Humans	Card turned	Response
89%	Vowel (A)	Correct!
62%	Even number (2)	Unnecessary!
25%	Odd number (7)	Correct!
16%	Consonant (D)	Unnecessary!

Observation 1': The Deontic Case [CG]

Again 4 cards; on one side person's age/backside drink.

If a person is drinking beer, then the person must be over 19 years of age.

Which cards must be turned to prove that the conditional holds?

beer	coke	22yrs	16yrs

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	beer	coke	22yrs	16yrs
Experimental Results	95%	2.5%	2.5%	80%

- Isomorphic to the previous problem. But, most get it right!
- Observations:
  - Humans can reason classically logically, but not always
  - Even for isomorpic problems human reasoning is not equivalent

Meta-analysis of WST [RKJL18]

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  - Rules: if p, then q; every p
  - Individual selection patterns (No aggregation!)
  - At least the four canonical selections: p, pq, $p\bar{q}$, $pq\bar{q}$ per Ss
- Inclusion of 228 experiments with N = 18,000 Ss:
  - Abstract: 104 exp; Everyday: 44 exp; Deontic: 80 exp

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- Aggregated results for the canonical selections in %

	p	pq	$pq\bar{q}$	$p\bar{q}$
Abstract	36	39	5	19
Everyday	23	37	11	29
Deontic	13	19	4	64

Data: `https://www.cc.uni-freiburg.de/data/`

Observation 2a: Belief Bias [EBP83]

All frenchmen drink wine
Some wine drinkers are gourmets

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Although the argument is widely accepted, it is not valid!

All frenchmen drink wine
Some wine drinkers are italians

Some frenchmen are italians

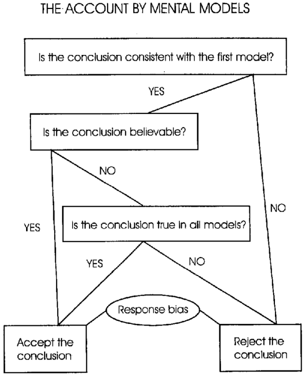
- Belief (in conclusion) Bias Effect!

Observation 2: Belief Bias – Meta-Analysis

Conclusion	Syllogism	
	Believable	Unbelievable
Valid	No cigarettes are inexpensive. Some addictive things are inexpensive.	No addictive things are inexpensive. Some cigarettes are inexpensive.
	Therefore, some addictive things are not cigarettes.	Therefore, some cigarettes are not addictive.
	P("valid") = 92%	P("valid") = 46%
Invalid	No addictive things are inexpensive. Some cigarettes are inexpensive. Therefore, some addictive things are not cigarettes.	No cigarettes are inexpensive. Some addictive things are inexpensive. Therefore, some cigarettes are not addictive.
	P("valid") = 92%	P("valid") = 8%

Example and numbers taken from [TKS+18].

Belief Bias – Meta-Analysis [TKS+18]



Can be explained by

- Background knowledge
- Erroneously reasoning about consistency instead of deductive reasoning
- Humans focusing on the conclusion instead on the reasoning process

Picture from [KMN00]

- Data can be found here: `https://osf.io/8dfyv/`

Observation 2: Knowledge frame [TK83]

Linda is 31 years old, single, outspoken and very intelligent. As a student she concerned herself thoroughly with subjects of discrimination and social justice and participated in protest against nuclear energy.

Rank the following statements by their probabilities.

- Linda works as a bank teller.
- Linda works as a bank teller and is an active feminist.

- Result: More than 80% judge Linda works as a bank teller and is an active feminist to be more likely than Linda works as a bank teller.
- BUT: $p(a \wedge b) \leqslant p(a)$ or $p(b)$
- Hence, most answer falsely from the perspective of probability!
- Instead humans use the so called representativity heuristic.

Observation 3: Nonmonotonicity

- If Lisa has an essay to write, Lisa will study late in the library
- If the library is open, Lisa will study late in the library
- Lisa an essay to write
  - 🔴 Lisa will study late in the library
  - 🟡 Nothing follows
  - 🟢 Can't say or I have another solution

The Suppression Task [Byr89]

- *If she has an essay to write, she will study late in the library.*

- *She has an essay to write.*

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95% of all subjects conclude (modus ponens):

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Only 60% of all subjects conclude:

- She will study late in the library.

A logic is called non-monotonic if the set of (logical) conclusions from a knowledge base is not necessarily preserved when new information is added to the knowledge base.

- Everyday reasoning is often non-monotonic [SVL08, JL06]

Suppression Task

Facts	Conditional
	If she has an essay to finish, then she will stay late in the library
She has an essay to finish	She will study late in the library (96% L)

Suppression Task

Facts	Conditional	Alternative Argument
	If she has an essay to finish, then she will stay late in the library	If she has a textbook to read, then she will stay late in the library
She has an essay to finish	She will study late in the library (96% L)	She will study late in the library (96% L)

Suppression Task

Facts	Conditional	Alternative Argument	Additional Argument
	If she has an essay to finish, then she will stay late in the library	If she has a textbook to read, then she will stay late in the library	If the library stays open, then she will stay late in the library
She has an essay to finish	She will study late in the library (96% L)	She will study late in the library (96% L)	She will study late in the library (38% L)

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She does not have an essay to finish	She will not study late in the library (46% $\neg L$)	She will not study late in the library (4% $\neg L$)	She will not study late in the library (63% $\neg L$)

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Suppression Task: Classical Logic

If she has an essay to finish	then she will stay late in the library	$l \leftarrow e$
If she has a textbook to read	then she will stay late in the library	$l \leftarrow t$
If the library stays open	then she will stay late in the library	$l \leftarrow o$

Clauses	Facts	Classical Logic
$l \leftarrow e$	e	$\models l$
$l \leftarrow e \quad l \leftarrow t$	e	$\models l$
$l \leftarrow e \quad l \leftarrow o$	e	$\models l$
$l \leftarrow e$	$\neg e$	$\not\models \neg l$
$l \leftarrow e \quad l \leftarrow t$	$\neg e$	$\not\models \neg l$
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For more see [DHR12].

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$l \leftarrow e$	$\neg e$	$\not\models \neg l$	46% $\neg L$	Denial of the Antecedent
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Classical logic does not adequately represent the suppression task.

For more see [DHR12].

Intermediate summary

- Instead of analyzing aggregated values, single responses provide the "real" inference process.
  $\Rightarrow$ Always look at the RAW data of an individual human
- Human reasoners generate patterns that can not be reproduced by classical logic.
- Some answer patterns have implications for other answer patterns (see, [RKJL18]).
- Three-valued approaches are required [RDKH16].

Formal inference methods

Do formal nonmonotonic inference approaches show this behavior?

- Change of perspective:
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- There are many nonmonotonic formalisms, e.g.,
  - System P
  - System Z
  - Reiter Default Logic
  - c-Representations
  - c-Representations + Revision
  - Logic Programming with Weak Completion Semantics

$\Rightarrow$ See Section 3

Talk Overview

Section 2

Formal models of commonsense reasoning

The relevance of uncertain reasoning

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Costly or even disastrous consequences may result from ignoring uncertainty.

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$\dfrac{Penguin \models Bird}{Penguin \wedge Black \models Bird}$ *Penguins are birds.* / *Black penguins are birds.* :)

$\dfrac{Bird \models Fly}{Bird \wedge Penguin \models Fly}$ *Birds can fly.* / *Penguin-birds can fly.* :(

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$\Rightarrow$ Logics without monotonicity

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Commonsense reasoning $\mid\!\sim$

Inference operator

$$\begin{aligned} C &: 2^{\mathcal{L}} \to 2^{\mathcal{L}} \\ C(\mathcal{F}) &= \{G \in \mathcal{L} \mid \mathcal{F} \mid\!\sim G\} \end{aligned}$$

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Tweety the penguin

Birds fly, penguins are birds, but penguins don't fly

$$bird \mid\!\sim fly, penguin \wedge bird \mid\!\sim \neg fly$$

Basic strategies of (nonmonotonic) commonsense reasoning

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Basically, two types of rules are used:

- Rules with default assumptions: Reiter's default logic, answer set programming, weak completion semantics
- Defeasible rules: Conditional reasoning, Poole's default logic

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(α) *If she has an essay to write, then she will study late in the library* and $(e \rightarrow l)$
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62% concluded that: *She may or may not study late in the library.*

We call this the suppression effect.

Reiter's default rules

Let φ, $\psi_1, \ldots, \psi_n$ and χ be (classical) formulas.

Reiter default rule

$$\delta = \frac{\varphi \ : \ \psi_1, \ldots, \psi_n}{\chi}$$

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$$\delta = \frac{\varphi \ : \ \psi_1, \ldots, \psi_n}{\chi}$$

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If φ is known, and $\psi_1, \ldots, \psi_n$ can be consistently assumed (i.e., none of $\neg\psi_i$ is known), then conclude χ.

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$$C_{\Delta}^{Reiter}(W) = \{\phi \mid W \mid\!\sim_{\Delta}^{Reiter} \phi\}$$

is the corresponding inference operator.

Reiter and Suppression task [Byrne 1989; Ragni, Eichhorn, Kern-Isberner 2016]

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where ab_1 is an *abnormality predicate* which expresses that nothing abnormal is known.

Process tree: Suppression Task [Ragni, Eichhorn, Kern-Isberner 2016]

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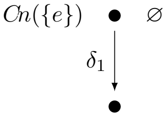

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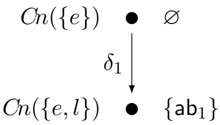

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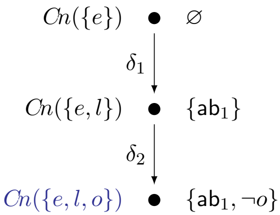

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$\downarrow \delta_1$

$Cn(\{e, l\}) \quad \bullet \quad \{\mathsf{ab}_1\}$

$\downarrow \delta_2$

$Cn(\{e, l, o\}) \quad \bullet \quad \{\mathsf{ab}_1, \neg o\}$

$Cn(\{e, l, o\})$ is the only extension of this theory, thus we have

$$e \mid\!\sim_{\Delta}^{Reiter} l,$$

i.e., no suppression effect.

Extended logic programming/Answer set programming

Extended logic program

An extended logic program $\mathcal{P}$ is a set of rules

$$r : \quad H \leftarrow A_1, \ldots, A_n, not\ B_1, \ldots, not\ B_m.$$

with literals $H, A_1, \ldots, A_n, B_1, \ldots, B_m$ and default negation not.

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Model the Suppression Task [Dietz et al. 2012]

(α) *If she has an essay to write, then she will study late in the library* and $(e \rightarrow l)$
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In logic programming, the (commonsense) implication "if she has an essay to finish, she will study late in the library" could be encoded by the clauses

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Now apply the weak completion semantics ...

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Weak completion semantics shows the suppression effect.

Defeasible Rules, Conditionals & Inference relation

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- Based on calculi/preferential structures, i.e.
  System C, System P [Kraus, Lehmann & Magidor 1990]

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$$\Delta^{\mathsf{supp},1} = \{\ (l|e),\ (l|o)\ ,\ (e|\top)\ \}$$
$$\Delta^{\mathsf{supp},2} = \{\ (l|e),\ (\bar{l}|\bar{o}),\ (e|\top)\ \}$$

Ranking functions and conditionals

Ordinal conditional functions (OCF, ranking functions[2]) [Spohn 1988]
$\kappa : \Omega \to \mathbb{N}(\cup\{\infty\})$ (Ω set of possible worlds, $\kappa^{-1}(0) \neq \varnothing$)

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Validating conditionals

$$\kappa \models (B|A) \text{ iff } \kappa(AB) < \kappa(A\overline{B})$$

κ accepts a conditional $(B|A)$ iff AB (its verification) is more plausible than $A\overline{B}$ (its falsification).

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Ranking functions – example

Example (essay)

$\kappa(\omega) = 4$	$e\bar{l}\,o$
$\kappa(\omega) = 2$	$\bar{e}\,lo \quad \bar{e}\,\bar{l}\,\bar{o}$
$\kappa(\omega) = 1$	$\bar{e}\,l\bar{o} \quad el\bar{o}$
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System Z [Pearl 1990]

For $\Delta = \{(B_1|A_1), \ldots, (B_n|A_n)\}$ build a partitioning $(\Delta_0, \Delta_1, \ldots, \Delta_k)$,

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κ^z imposes penalty points on the worlds for falsifying conditionals.

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- The partitioning is ordered and unique.

This gives us the unique ranking function κ^z defined by

$$\kappa^z(\omega) = \begin{cases} 0, & \text{if } \omega \text{ does not falsify any conditional in } \Delta \\ 1 + \max\{i \mid \omega \models A\overline{B} \text{ for some } (B|A) \in \Delta_i\}, & \text{otherwise} \end{cases}$$

κ^z imposes penalty points on the worlds for falsifying conditionals.

Z-inference wrt Δ, $\mid\!\sim^z_\Delta$, is defined as follows:

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System Z and the Suppression Task

$$\Delta^{\text{supp},1} = \{\ (l|e),\ (l|o)\ ,\ (e|\top)\ \}$$
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Yielding κ_1^z, κ_2^z:

ω	$\kappa_1^z(\omega)$	$\kappa_2^z(\omega)$	ω	$\kappa_1^z(\omega)$	$\kappa_2^z(\omega)$
elo	0	0	$\overline{e}lo$	1	1
$el\overline{o}$	0	2	$\overline{e}l\overline{o}$	1	2
$e\bar{l}o$	1	1	$\overline{e}\bar{l}o$	1	1
$e\bar{l}\,\overline{o}$	1	1	$\overline{e}\bar{l}\,\overline{o}$	1	1

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$e\bar{l}\,\bar{o}$	1	1	$\bar{e}\bar{l}\,\bar{o}$	1	1

In both cases we gain no suppression effect, $\top \mid\!\sim^z_{\Delta^{\text{supp},i}} l$, hence $\kappa_i^z \models (l|\top)$.

C-representations [Kern-Isberner 2001]

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c-representation of $\Delta = \{(B_1|A_1), \ldots, (B_n|A_n)\}$ is defined by

$$\kappa_\Delta(\omega) = \sum_{\omega \models A_i \overline{B_i}} \kappa_i^-$$

with parameters $\kappa_1^-, \ldots, \kappa_n^- \in \mathbb{N}_0$ chosen such that

$$\kappa_\Delta \models (B_j|A_j), 1 \leqslant j \leqslant n,$$

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holds. Computation of c-representations can be characterised by a CSP:

$$\kappa_j^- > \min_{\omega \models A_j B_j} \sum_{\substack{i \neq j \\ \omega \models A_i \overline{B_i}}} \kappa_i^- - \min_{\omega \models A_j \overline{B_j}} \sum_{\substack{i \neq j \\ \omega \models A_i \overline{B_i}}} \kappa_i^-$$

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C-representation are derived from general principles of change and differ from System Z, e.g., they do not have the drowning problem.

Mimicking weak completion semantics

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So, if we encode this in the knowledge base

$$\Delta_3 = \{(l|eo), (e|\top)\},$$

then we find both for system Z and c-representations that

$$Bel(\kappa_3^z) = Bel(\kappa_3^c) = Cn(elo \vee e\bar{l}\bar{o}),$$

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which means that l is no longer believed!

Occurrence of the suppression effect depends more on the modelling than on the chosen method!

System P [Kraus, Lehmann & Magidor 1990]

System P consists of the following rules:
Reflexivity, Left Logical Equivalence, Right Weakening, Cut, Or, Cautious Monotony

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The P stands for preferential because System P can be characterised by preferential models.
However, another useful characterisation is due to Goldszmidt and Pearl:

Proposition [Goldszmidt and Pearl, 1996]
$A \mid\!\sim^{p}_{\Delta} B$ if and only if $\Delta \cup \{(\overline{B}|A)\}$ is inconsistent[3].

[3] Consistency is given if there is a System Z partitioning.

System P and the Suppression Task [Ragni, Eichhorn, Kern-Isberner 2016]

The two modellings for the suppression task:

$$\Delta_1^{\mathsf{supp}} = \{\ (l|e),\ (l|o)\ ,\ (e|\top)\ \}$$
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$$\top \mid\!\sim^{p}_{\Delta_1^{\mathsf{supp}}} l \qquad\qquad \top \mid\!\sim^{p}_{\Delta_2^{\mathsf{supp}}} l$$

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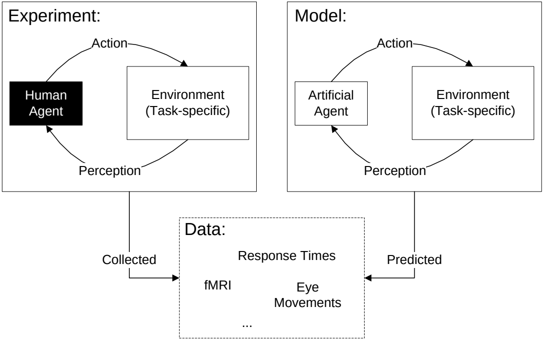
This is surprising since System P is often considered as very well-behaving nonmonotonic formalism.

Talk Overview

Section 3

Cognitive aspects of Cognitive Logics

What does a cognitive model do?



- Reconstructive and generative models (Lüer & Spada, 1990):
  - Reconstructive: Conceptualising structures and processes that underly mental activity
  - Generative: The execution of a model not only describes psychological phenomena but also generates them
    $\Rightarrow$ Compare model predictions with empirical data

Phases of cognitive modeling

Four phases can be considered (e.g., Lewandowski & Farrell, 2011):

1. Task analysis:
    - What knowledge is needed to solve a task?
    - What are processes involved in generating the knowledge to solve a task
    - What are relevant structures an architecture used to specify a model?

2. Empirical data
    - Reconstruction of trace/statistical measure for one participant
    - Reconstruction of some statistical measure which considers all participants

Phases of cognitive modeling

3. Model implementation
    - Architecture selection (e.g. Neural Network, MPT, Logic)
    - Process specification
    - Parameter estimation (e.g. simulated annealing, maximum likelihood estimation)

4. Model validation
    - Parameter uncertainty
    - Model comparison
    - Model interpretation

$\Rightarrow$ Mental representation ($\rightarrow$ conditionals) and the inference mechanism are core issues

How can we evaluate cognitive theories?

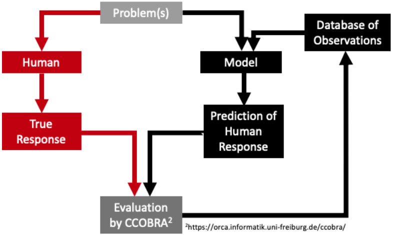
Simon and Wallach (1999) require a generative theories to have:

- Product correspondence: this requires that the cognitive model shows a similar overall performance as human data
- Correspondence of intermediate steps: this requires that assumed processes and steps in the model parallels separable stages in human processing
- Error correspondence: this requires that the same error patterns in the model emerge than in experimental data
- Correspondence of context dependency: this is a comparable sensitivity to known external influences

Syllogistic Reasoning: Aggegate Data

- Existing cognitive theories cover the Most Frequent Answer on aggregate data, e.g., mReasoner and WCS more than 94% [CDHR16]
- They can even be improved by additional heuristics, e.g., when does someone responds "nothing follows"? [RBDR20, RDBR19]
- But, if we want to have an AI assistant that can adapt to our reasoning capabilities, does modeling "group answers" really helps us?

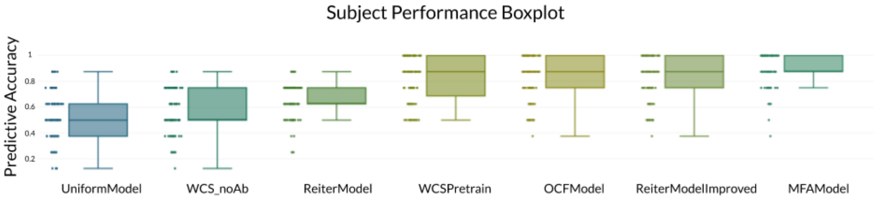
Cognitive Computation for Behavioral Reasoning Analysis (CCOBRA)



- Benchmarking tool integrating *individual* in prediction loop
- Models are evaluated based on their predictive accuracies
- CCOBRA offers pretrain, adapt, and predict methods
- Applied to syllogistic, relational, propositional reasoning [RBR20, RFB+19]

`https://orca.informatik.uni-freiburg.de/ccobra`

Nonmotonic Logics . . .



- Abduction in WCS is relevant
- Reiter with modus tollens and affirmation of consequence lead to ReiterModelimproved
- OCF performs identical to ReiterModelImproved

Summary and new questions

- Humans deviate from valid inferences by classical logic, but **nonmonotonic logics** are competitive.
- The extended version of Reiter's model is a functionally equivalent model to the OCF.
- Pre-trained WCS only slightly worse than Reiter Model Improved and OCF $\rightarrow$ missed **MP** predictions due to abnormalities, but, in contrast to them, successfully models **DA** by abduction.
- Decrease of predictive performance of WCS by almost 26% when not using abduction.
- Individualization relevant in all other problems relevant as well, e.g., in Wason Selection Task [RKJL18, BIMR19], etc.

Talk Overview

Section 4

Future Challenges

You can make the difference!

- There exist many more reasoning problems in cognitive psychology
  - The need for a set of benchmark arises
- There are many logics and reasoning formalisms in AI
  - The need for implementations in a testable framework arises
  - and *the core point* is logics need to be made adaptive (or dynamic) that based on observations they can adapt in explain *black box processes*
- Ultimate goal: Cognitive logics are white-boxing the black-box process of individual human reasoning

Cognitive Logics Website

http://cognitive-logics.org/

Talk Overview

Section 5

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